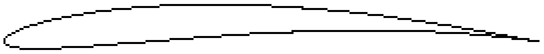
EAE 127 Project 2



Thin Airfoil Theory

Fall 2009 Due Monday, 10/12/09 (in drop box C)

The objective of this project is to get familiar with the thin-airfoil theory and also the code *thinair.f* to analyze the inviscid, incompressible, small disturbance flow past thin profiles. A thin airfoil is represented by a camber line $d(x)$ proportional to dm with $d(0) = d(1) = 0$ and a thickness distribution $e(x)$ proportional to em with $e(0) = e(1) = 0$. Note that x, z and other lengths are nondimensionalized by the chord cm. The incidence α (in *radian*) is simulated by a shearing transformation so that the upper and lower surfaces of the airfoil are defined by equation (1).

$$z_u(x) = d(x) + 0.5e(x) + \alpha \cdot (1 - x) \qquad 0 \le x \le 1 \qquad (1)$$
$$z_o(x) = d(x) - 0.5e(x) + \alpha \cdot (1 - x)$$

The fundamental equation of thin airfoil theory states that the flow is tangent to the camber line :

$$w(x,0) = -\frac{1}{2\pi}\int_0^1 \frac{\Gamma'(\xi)d\xi}{x - \xi} = U\left(d'(x) - \alpha\right) \qquad (2)$$

where $d'(x)$ represents the slope of the camber line and $w(x,0)$ represents the velocity perturbation in the z-direction. Equation (2) is solved for Γ subject to the Kutta-Joukowski condition $\Gamma'(cm) = \Gamma'(1) = 0$.

Equation (2) is solved numerically using control points along the x-axis according to

$$x_i = \frac{c}{2}\left(1 - \cos(\Theta_i)\right) \quad , \quad i = 1,\dots,ix \quad , \quad \Theta_i = (i-1)\Delta\Theta \quad , \quad \Delta\Theta = \frac{\pi}{ix - 1}$$

Γ_i, u_i, w_i are computed at the control points. Dummy integration points are placed between the control points as

$$\xi_j = \frac{c}{2}\left(1 - \cos\left(\Theta_j + \Delta\Theta/2\right)\right) \qquad j = 1,\dots,ix-1$$

$\Gamma'_j = (\Gamma_{j+1} - \Gamma_j)/(x_{j+1} - x_j)$ corresponds to the infinitesimal strength of the vortex located at ξ_j. The integral is replaced by a summation and the system for the Γ_j 's reads :

$$-\frac{1}{2\pi}\sum_{j=1}^{ix-1}\frac{\Gamma_{j+1} - \Gamma_j}{x_i - \xi_j} = U\left(d'_i - \alpha\right) \qquad i = 2,\dots,ix-1$$
$$\Gamma_1 = 0$$

$$\Gamma_{ix} = \left(\frac{\left(x_{ix} - x_{ix-1}\right)^2}{\left(x_{ix} - x_{ix-2}\right)^2} \Gamma_{ix-2} - \Gamma_{ix-1} \right) \Big/ \left(\frac{\left(x_{ix} - x_{ix-1}\right)^2}{\left(x_{ix} - x_{ix-2}\right)^2} - 1 \right)$$

The last equation is the K-J condition. This linear system for the Γ_i's is solved iteratively (relaxation method) in about 1000 iterations.

The small disturbance pressure coefficient is given by $C_{p,sd} = -2u/U$. Note that the small disturbance velocity field (u, w) has been also been "regularized" to account for the leading edge singularity in the case of a blunt nose airfoil. Both regularized and thin airfoil theory results are outputted in the *thinair.xcp*, *thinair.xu* and *thinair.xw* files.

1. For the following cases, plot the airfoil shape, the surface pressure coefficients $C_{p,sd}$ and C_p, the local vortex strength $\Gamma'(x)$ and the partial circulation $\Gamma(x)$.

(a) flat plate	(α = 2º)
(b) parabolic plate	($dm/cm = 0.086$) (α = 0º)
(c) symmetric Q-J airfoil	($em/cm = 0.12$) (α = 2º)
(d) cambered Q-J airfoil	($dm/cm = 0.086, em/cm = 0.12$) (α = 0º)

2. In (a)-(d) above, compare the thin airfoil solutions for C_p with the analytical solutions obtained using Project 1 code *qjairflow.f* (plot the two sets of C_p's on the same set of axis). Discuss your results. Also compare analytical solutions for C_p with the regularized C_p values for all 4 airfoils.

Discuss particularly the effects of camber, thickness, and incidence angle on lift, and the comparison between the exact theory of Project 1 and the thin airfoil theory of Project 2. What assumptions are made? Why is the Kutta condition important?