EAE 127 - MIDTERM 11/04/03

(Open Notes, open Book)

(Give unambiguous answers. Use results derived in Class)

1. Inviscid, Incompressible Flow (15 points)

In thin airfoil theory, consider a thin cambered plate of equation $z = d(x)$ that produces, at a particular incidence α_a, a vorticity distribution given by the third mode only:

$$\Gamma'[x(t)] = 2V_\infty A_3 \sin 3t$$

$$x(t) = \frac{c}{2}(1 - \cos t),\ 0 \leq t \leq \pi.$$

1.1 Global Coefficients

Give the value of the aerodynamic coefficients C_l, C_d and $C_{m,o}$ at that particular value $\alpha = \alpha_a$, according to thin airfoil theory, in terms of the corresponding values of $A_0,\ A_1, ...$
If α varies, how do these coefficients vary? (Hint: use the relation between α and the coefficients $A_0,\ A_1, ...$ in the expansion of $\Gamma'[x(t)]$).
If thickness is added to the thin cambered plate, how will these coefficients be affected?

1.2 Center of Pressure

Give the definition of the center of pressure.
Where is the center of pressure located for this airfoil?

1.3 Geometry of the Thin Cambered Plate

Give the relation between the slope of the thin cambered plate $d'(x)$ and the Fourier series of cosines. Using the chain rule, $\frac{dd[x(t)]}{dt} = d'[x(t)]\frac{dx(t)}{dt}$, integrate this equation and find A_0 and $d[x(t)]$, given that $d[x(0)] = d[x(\pi)] = 0$ (use the identity $\cos 3t = 4\cos^3 t - 3\cos t$). Sketch the thin cambered plate.

1.4 Result

What is the incidence of adaptation (ideal angle of attack)?
Sketch the vorticity $\Gamma'[x(t)]$ at ideal angle of attack.

2. Linearized Supersonic Flow (10 points)

The above profile equips the horizontal tail of a supersonic aircraft flying at $M_0 > 1$. Let $\beta = \sqrt{M_0^2 - 1}$.

2.1 Global Coefficients

Give the expressions of the following aerodynamic coefficients C_l, C_d and $C_{m,o}$, in terms of β and the camber $d(x)$ at $\alpha = 0$ (you are not asked to compute them).
If thickness were added to the cambered plate, which of these coefficients would be affected?

2.2 Moment Coefficient

Using the variable t , $x(t) = \frac{c}{2}(1 - \cos t)$ and the identity $\cos 3t = \cos t\,(1 - 4\sin^2 t)$, integrate the above formula for $(C_{m,o})_{\alpha=0}$.

2.3 Static Stability

Write the aerodynamic moment coefficient, $C_{m,D}$, about an arbitrary point x_D in terms of $(C_{m,o})_{\alpha=0}$, α, β and $\frac{x_D}{c}$. Show that the airfoil will be stable about its equilibrium incidence if $(1 - \frac{2x_D}{c}) > 0$.