EAE 127 - MIDTERM 11/03/06

(Open Notes, open Book)

(Give unambiguous answers. Use results derived in Class)

1. Inviscid, Incompressible Flow (20 points)

1.1 Flap Modelization with Thin Airfoil Theory

A thin parabolic plate has a hinge located at (x_f, z_f), $0 \leq x_f \leq c$, about which the end of the plate can rotate by an angle δ_f (positive down) and that plays the role of a flap. The equation of the thin plate is given by:

$$f^{\pm}(x) = d(x) = \begin{cases} 4d\frac{x}{c}(1-\frac{x}{c}),\ 0 \leq x \leq x_f \\ 4d\frac{x}{c}(1-\frac{x}{c}) + \delta_f(x_f - x),\ x_f \leq x \leq c \end{cases}$$

See Fig. 1. Calculate the slope $d'(x)$ of the plate as a function of x, then as a function of t (check your algebra as other results depend on this), given that

$$x(t) = \frac{c}{2}(1 - \cos t),\ 0 \leq t \leq \pi$$

1.2 Coefficient A_0

Let t_f be the value of the parameter t corresponding to the flap hinge in the parametric representation of the thin plate as

$$x_f = x(t_f) = \frac{c}{2}(1 - \cos t_f),\ 0 \leq t_f \leq \pi$$

Using the formula derived in class $A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi} d'[x(t)]dt$, calculate A_0. Check your result for $t_f = \pi$ ($x_f = c$). Conclude. Check your result for $t_f = 0$ ($x_f = 0$). Conclude.
What is the angle of adaptation $\alpha_{adapt}(t_f, \delta_f)$? Check your result for $t_f = \pi$ ($x_f = c$). Conclude. Check your result for $t_f = 0$ ($x_f = 0$). Conclude.
Calculate the incidence of adaptation for $x_f = 3c/4$ and $\delta_f = 10\ deg$.
Sketch the plate and, qualitatively, the flow at the calculated incidence of adaptation, in particular the streamlines near the leading edge and the trailing edge .

1.3 Coefficient A_1

Calculate A_1 in terms of the average camber $\frac{d}{c}$, t_f and δ_f. Check your result for $t_f = \pi$ ($x_f = c$). Conclude.

1.4 Coefficients $A_2, ..., A_n,\ n \geq 2$

Calculate the general term A_n and give the value of A_2.

1.5 Global Coefficients

Give the value of the aerodynamic coefficients C_l, C_d and $C_{m,o}$ in terms of α, t_f and δ_f.
If thickness is added to the thin cambered plate, how will these coefficients be affected?

1.6 Aerodynamic Center (Bonus Points)

Give the definition of the *aerodynamic center*.
Find the moment at the aerodynamic center.

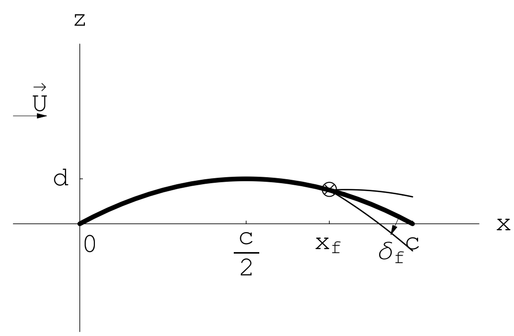

Figure 1: Parabolic cambered plate with flap

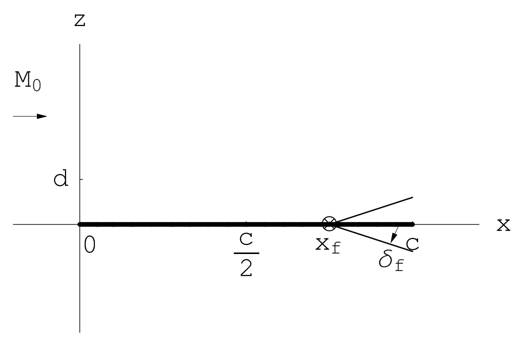

Figure 2: Flat plate with flap

2. Linearized Supersonic Flow (10 points)

2.1 Flap Modelization with Thin Airfoil Theory

Consider the flat plate with flap which represents the tail of a supersonic aircraft flying at $M_0 > 1$. Let $\beta = \sqrt{M_0^2 - 1}$. See Fig. 2.
The equation of the tail is

$$\begin{cases} d(x) = 0, \ 0 \leq x \leq x_f \\ d(x) = (x_f - x)\delta_f, \ x_f \leq x \leq c \end{cases}$$

Calculate the slope $d'(x)$ of the tail as a function of x (two expressions).
Calculate the pressure coefficients $C_p^+(x)$ and $C_p^-(x)$ along the tail in terms of β, α and δ_f(two expressions for each).
Plot on a graph $-C_p^+(x)$ and $-C_p^-(x)$. The area between the curves represents the lift coefficient.

2.2 Global Coefficients: C_l

Calculate the aerodynamic coefficients C_l in terms of x_f, β, α and δ_f. Hint: go back to the definition of lift and integrate the C_p's to get C_l. This is because a flap setting δ_f is equivalent to a change in incidence $\delta\alpha = (1 - x_f/c)\delta_f$.

2.3 Global Coefficients: C_d

Calculate C_d in terms of x_f, β, α and δ_f.