EAE 127 - MIDTERM 11/02/07

(Open Notes, open Book)

(Give unambiguous answers. Use results derived in Class)

1. Inviscid, Incompressible Flow (20 points)

A thin airfoil design is considered for which the vorticity distribution is given by the second Fourier mode only. The purpose of the analysis is to find the airfoil shape and the local and global aerodynamic coefficients.

1.1 Second Mode Airfoil Shape

$$\begin{cases} \Gamma'[x(t)] = 2UA_2 \sin 2t, \ A_2 > 0 \\ x(t) = \frac{c}{2}(1 - \cos t), \ 0 \leq t \leq \pi \end{cases}$$

Sketch Γ'. Show that $d'[x(t)]$ is given by (Hint: use the identity $\cos 2t = 2\cos^2 t - 1$)

$$d'(x) = \alpha - A_0 + A_2 \left(1 - 8\frac{x}{c} + 8(\frac{x}{c})^2\right)$$

Upon integration, find the expression for A_0 and write the equation for $d(x)$. Sketch the camberline (Note that A_2 is an arbitrary multiplicative constant).

1.2 Second Mode Aerodynamic Characteristics

Give the *incidence of adaptation* (ideal angle of attack). Make a sketch of the flow in this case, showing in particular the streamlines in the leading edge and trailing edge regions.
Give the lift coefficient $C_l(\alpha)$ as a function of α. What is the value of C_l when the leading edge is adapted? What is the value of C_l when $\alpha = 0$?
Give the moment coefficient $C_{m,o}(\alpha)$ as a function of α. Using the change of moment formula, find the moment $C_{m,\frac{c}{2}}(\alpha)$ at mid-chord as a function of α.

1.3 Static Stability About an Axis

Find the incidence of equilibrium, α_{eq}, if the profile can rotate without friction about an axis located at mid-chord (neglect weight). Study the static stability of the profile at $\alpha = \alpha_{eq}$, and indicate whether the equilibrium is stable or unstable.

2. Linearized Supersonic Flow (10 points)

2.1 Pressure Distribution on a Thin Cambered Plate

Let $\beta = \sqrt{M_\infty^2 - 1}$, $M_\infty > 1$. Consider a thin cambered plate of equation:

$$d(x) = Ac\frac{x}{c}\left(1 - 2\frac{x}{c}\right)\left(1 - \frac{x}{c}\right)$$

where A is an arbitrary constant ($A > 0$). Calculate the slope $d'(x)$ as a function of x.
Calculate the pressure coefficients $C_p^+(x)$ and $C_p^-(x)$ along the plate in terms of β, for $\alpha = 0$.
Plot on a graph $-C_p^+(x)$ and $-C_p^-(x)$.

2.2 Global Coefficients: C_l, C_d, $C_{m,o}$

Give the aerodynamic coefficient $C_l(\alpha)$ and calculate the aerodynamic coefficients $(C_d)_{\alpha=0}$, $(C_{m,o})_{\alpha=0}$, for $\alpha = 0$.

2.3 Static Stability About an Axis

Calculate $C_{m,\frac{c}{2}}(\alpha)$, using the change of moment formula. If the profile can rotate freely about an axis placed at mid-chord, find α_{eq}, the equilibrium angle. Is the equilibrium statically stable, neutral or unstable?