EAE 127 - MIDTERM 11/02/09

(Open Notes, open Book)

(Give unambiguous answers. Use results derived in Class)

1. Inviscid, Incompressible Flow (20 points)

Design of a Thin Cambered Plate: We wish to design the *simplest* cambered plate with a hinge located at the leading edge. The hinge is clamped and the thin airfoil will rotate without friction to an equilibrium position corresponding to a specified lift coefficient $C_{l,eq}$ such that the leading edge is "adapted". We will neglect the weight of the plate. By "simplest" plate we mean a plate that has the smallest possible number of non-zero Fourier coefficients $A_1, A_2, ...A_n$ for Γ'.

1.1 Definition of the Center of Pressure

Give the definition of the *Center of Pressure*.

1.2 Condition for Equilibrium

Give the condition for equilibrium, given that the plate can rotate freely about its leading edge (value of force, moment?).
Find the condition on the Fourier coefficients to satisfy this equilibrium requirement.

1.3 Leading Edge "Adaptation"

Explain what is meant by leading edge "adaptation".
How does this condition translate for the Fourier coefficients?
Accounting for adapted leading edge, what can you say of the coefficients for the simplest plate?

1.4 Lift Coefficient at Equilibrium

Give the expression of the lift coefficient C_l in terms of the Fourier coefficients.
What choice of the Fourier coefficients will give $C_l = C_{l,eq}$?

1.5 Equation of the Cambered Plate

Find the equation of the cambered plate. Hint: use the equation

$$d'[x(t)] = \alpha - A_0 + \sum_{n=1}^{\infty} A_n \cos nt$$

Keep the term A_0 and the other non-zero terms that satisfy the above requirements in the integration and use the identities

$$\cos t = 1 - \frac{2x}{c}, \quad \cos 2t = 1 - \frac{8x}{c} + \frac{8x^2}{c^2}$$

Find A_0.
Write the equation of the plate as a product of first degree terms. Hint: you know already two roots of the polynomial.

1.6 Plate Aerodynamic Coefficients

Give the expressions for the plate's C_l, $C_{m,o}$ and C_d.
What is the angle of adaptation or *ideal angle of attack* for this plate?
Check your results by calculating $C_l(\alpha_{adapt})$ and $C_{m,o}(\alpha_{adapt})$.

2. Linearized Supersonic Flow (10 points)

Let $\beta = \sqrt{M_\infty^2 - 1}$, $M_\infty > 1$. Consider a thin cambered plate of equation

$$d(x) = d\left(11\frac{x}{c} - 27\frac{x^2}{c^2} + 16\frac{x^3}{c^3}\right)$$

d is related to the camber ($d > 0$).

2.1 Pressure Distribution on the Thin Cambered Plate

Calculate the pressure coefficients $C_p^+(x)$ and $C_p^-(x)$ along the plate, for $\alpha = 0$.
Plot on a graph $-C_p^+(x)$ and $-C_p^-(x)$.

2.2 Global Coefficients: C_l, $C_{m,o}$

Give the lift coefficient $C_l(\alpha)$.
Calculate the aerodynamic coefficient $(C_{m,o})_{\alpha=0}$. Hint: it is convenient to use $\xi = x/c$ as variable in the integration which now runs from zero to one.
Conclude by giving the aerodynamic moment coefficient $C_{m,o}(\alpha)$.

2.3 Static Equilibrium About an Axis

If the profile can rotate freely about an axis placed at the leading edge (neglect weight), find α_{eq}, the equilibrium angle and $C_{l,eq}$. Sketch the profile at equilibrium and indicate with a vector $\vec{C}_{l,eq}$ the resulting force and its point of application on your drawing.

2.4 Static Stability

Is the equilibrium statically stable, neutral or unstable?