# EAE 127 - MIDTERM/SOLUTION 11/04/03

## 1. Inviscid, Incompressible Flow (15 points)

### 1.1 Global Coefficients

Given the vorticity corresponding to the third mode only, the coefficients are $A_0 = A_1 = A_2 = 0$, $A_3 \neq 0$ and $A_4 = ... = A_n = 0$, $n \geq 4$.
According to the theory:
$C_l = 2\pi(A_0 + \frac{A_1}{2}) = 0$
$C_d = 0$ (always)
$C_{m,o} = -\frac{\pi}{2}(A_0 + A_1 - \frac{A_2}{2}) = 0$.
Only $A_0$ depends on $\alpha$ ($A_0 = \alpha + const$). Hence, the aerodynamics coefficients $C_l$ and $C_{m,o}$ will vary as given above in terms of $A_0$. $C_d$ remains zero in all cases.
If thickness is added to the thin cambered plate, none of these coefficients will be affected.

### 1.2 Center of Pressure

The center of pressure is the point about which the moment of the aerodynamic forces is zero.
The center of pressure for this airfoil is located at the quarter chord as
$\frac{x_{cp}}{c} = \frac{A_0}{4A_0} = \frac{1}{4}$.

### 1.3 Geometry of the Thin Cambered Plate

According to the theory, the slope of the thin cambered plate verifies:
$d'[x(t)] = \alpha - A_0 + \sum_{n=1}^{\infty} A_n \cos nt = \alpha - A_0 + A_3 \cos 3t$.
Integrating in $t$ one gets:
$d(t) = \int_{\pi}^{t}(\alpha - A_0 + A_3 \cos 3t)\frac{c}{2}\sin t dt = \frac{c}{2}\int_{\pi}^{t}(\alpha - A_0 - 3A_3 \cos t + 4A_3 \cos^3 t)\sin t dt$
$= \frac{c}{2}[-(\alpha - A_0)\cos t + \frac{3}{2}A_3 \cos^2 t - A_3 \cos^4 t]_{\pi}^{t}$
$= \frac{c}{2}[-(\alpha - A_0)(\cos t + 1) + \frac{3}{2}A_3(\cos^2 t - 1) - A_3(\cos^4 t - 1)]$
Since $d(0) = 0$, one finds $A_0 = \alpha$ and the equation for the cambered plate reduces to:
$d(t) = A_3 \frac{c}{2}\sin^2 t\,(\frac{1}{2} - \sin^2 t)$.
The sketch of the thin cambered plate is shown in Fig. 1.

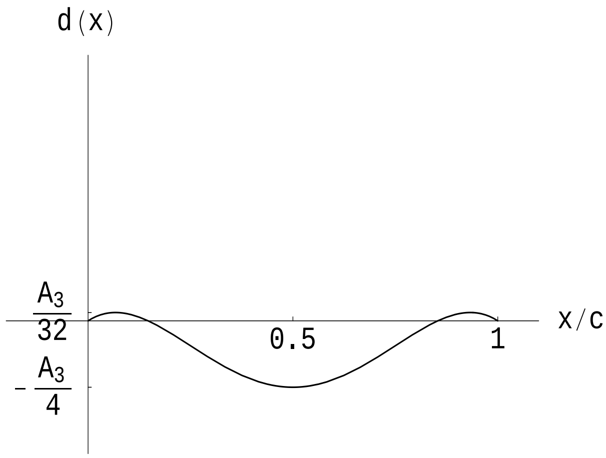


Fig. 1. Geometry of cambered plate

### 1.4 Result

The incidence of adaptation (ideal angle of attack) corresponds to $A_0 = 0$, i.e. the profile satisfies two Kutta-Joukowski conditions. Here this is the case when $\alpha_a = 0$.
The vorticity $\Gamma'[x(t)]$ at ideal angle of attack is sketched in Fig. 2.

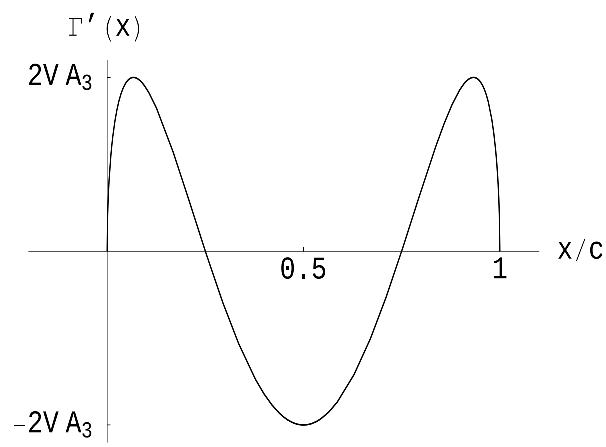


Fig. 2. Vorticity distribution

# 2. Linearized Compressible Flow (10 points)

## 2.1 Global Coefficients

The global coefficients are given by:
$C_l = \frac{4\alpha}{\beta}$
$C_d = \frac{4}{\beta}\int_0^c (d^{'}(x))^2 \frac{dx}{c} + \frac{4\alpha^2}{\beta}$
$C_{m,o} = \frac{4}{\beta}\int_0^c d^{'}(x)\frac{x}{c}\frac{dx}{c} - \frac{2\alpha}{\beta}$.
If thickness were added to the cambered plate, only the wave drag coefficient $C_d$ would be affected.

## 2.2 Moment Coefficient

We have seen that $d^{'}[x(t)] = A_3 \cos 3t$. Using the same change of variable as for subsonic flow, the moment coefficient reads:
$(C_{m,o})_{\alpha=0} = \frac{4A_3}{\beta}\int_0^\pi \cos 3t\, \frac{1}{2}(1-\cos t)\frac{1}{2}\sin t\, dt$
$= \frac{A_3}{\beta}\int_0^\pi (4\cos^3 t - 3\cos t)(1-\cos t)\sin t\, dt$
$= \frac{A_3}{\beta}\int_0^\pi (-4\cos^4 t + 4\cos^3 t + 3\cos^2 t - 3\cos t)\sin t\, dt$
$= \frac{A_3}{\beta}[\frac{4}{5}\cos^5 t - \cos^4 t - \cos^3 t + \frac{3}{2}\cos^2 t]_0^\pi = \frac{2}{5}\frac{A_3}{\beta}$.

## 2.3 Static Stability

The formula for the moment about point $D$ is:
$C_{m,D} = C_{m,o} + \frac{x_D}{c}Cl$.
Here we get:
$C_{m,D} = (C_{m,o})_{\alpha=0} - \frac{2\alpha}{\beta} + \frac{x_D}{c}\frac{4\alpha}{\beta} = (C_{m,o})_{\alpha=0} - \frac{2\alpha}{\beta}(1 - \frac{2x_D}{c})$.
At equilibrium $C_{m,D} = 0$. A small change of incidence gives:
$\Delta C_{m,D} = -\frac{2\Delta\alpha}{\beta}(1 - \frac{2x_D}{c})$
If the thin profile is in equilibrium and there is a perturbation that increases the incidence (nose up), stability requires that the moment be negative (nose down). This will be the case if $(1 - \frac{2x_D}{c}) > 0$.