EAE 127 - MIDTERM/SOLUTION 11/02/04

(Open Notes, open Book)

1. Inviscid, Incompressible Flow (20 points)

1.1 Design of a Thin Cambered Plate

In thin airfoil theory, *"leading edge adaptation"* means that the flow is smooth at the leading edge, in other words, it satisfies a Kutta-Joukowski condition there. Mathematically, this translates as $A_0 = 0$.
The design requirement:

$$C_{m,o} = -\frac{1}{2}C_l,$$

can be expressed in terms of the Fourier coefficients as

$$-\frac{\pi}{2}(A_0 + A_1 - \frac{A_2}{2}) = -\frac{1}{2}2\pi(A_0 + \frac{A_1}{2}).$$

This is one equation for three unknowns, however, at adaptation, $A_0 = 0$, hence it reduces to $A_2 = 0$.
The simplest cambered plate which satisfies this design requirement has the following Fourier coefficients:

$$A_0 = 0,\ A_1 \neq 0\ (arbitrary),\ A_2 = ... = A_n = 0,\ n \geq 2.$$

The tangency condition reads:

$$d^{'}[x(t)] = \alpha - A_0 + \sum_{n=1}^{\infty} A_n \cos nt = \alpha - A_0 + A_1 \cos t = \alpha - A_0 + A_1(1 - 2\frac{x}{c}).$$

Upon integration, say from $\xi = 0$ to $\xi = x$ one gets:

$$d(x) = (\alpha - A_0 + A_1)x - A_1\frac{x^2}{c}.$$

The condition $d(c) = 0$ yields $(\alpha - A_0)c = 0$, i.e. $A_0 = \alpha$.
Substituting back gives the equation of the camberline (parabolic plate)

$$d(x) = A_1 c\frac{x}{c}(1 - \frac{x}{c}).$$

The angle of adaptation corresponds to $A_0 = 0$, that is $\alpha_a = 0$.
The flow satisfies two Kutta-Joukowski conditions, as shown in Fig. 1.

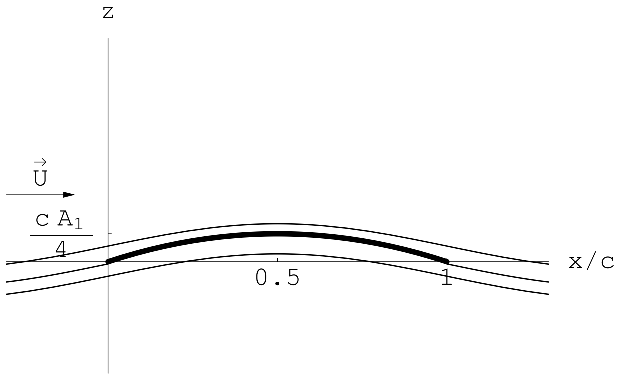

Fig.1. Sketch of streamlines at incidence of adaptation

1.2 Global Coefficients

At $\alpha = \alpha_a$, the aerodynamic coefficients are:

$$C_l = \pi A_1,\ Cd = 0,\ C_{m,o} = -\frac{\pi}{2}A_1 = -\frac{C_l}{2}.$$

Indeed, the requirement is satisfied.
At adaptation, the center of pressure is at $\frac{x_{cp}}{c} = -\frac{C_{m,0}}{C_l} = \frac{1}{2}$, the mid-chord. This is consistent with the symmetry of the flow with respect to the axis x=$\frac{c}{2}$.
If α varies, these coefficients become:

$$C_l = 2\pi(\alpha + \frac{A_1}{2}),\ C_d = 0,\ C_{m,o} = -\frac{\pi}{2}(\alpha + A_1).$$

The lift is a linear increasing function of α and the moment a linear decreasing function of α.
If thickness is added to the thin cambered plate, none of these coefficients will be affected.

1.3 Aerodynamic Center

The *aerodynamic center* is the point about which the moment of the aerodynamic forces is independent of α.
The moment at the aerodynamic center is

$$C_{m,ac} = -\frac{\pi}{4}A_1.$$

1.4 Result

The vorticity $\Gamma^{'}[x(t)]$ at ideal angle of attack is shown in Fig. 2.

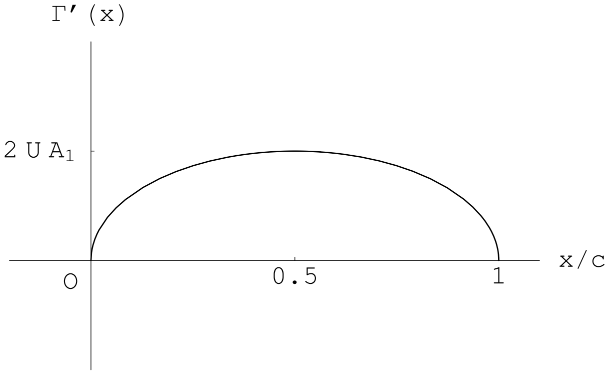

Fig.2. Vorticity distribution at incidence of adaptation

2. Linearized Supersonic Flow (10 points)

Consider the parabolic plate of equation $d(x) = A_1 c \frac{x}{c}(1 - \frac{x}{c})$ which represents the mean camberline of the wing of a supersonic aircraft flying at $M_0 > 1$. Let $\beta = \sqrt{M_0^2 - 1}$.

2.1 Global Coefficients at $\alpha = 0$

The aerodynamic coefficients C_l, C_d and $C_{m,o}$, at $\alpha = 0$ are:

$$C_l = 0,\ (C_d)_{\alpha=0},\ (C_{m,o})_{\alpha=0},$$

where (letting $\xi = x/c$):

$$(C_d)_{\alpha=0} = \frac{4}{\beta}\int_0^c [d^{'}(x)]^2 \frac{dx}{c} = \frac{4}{\beta}A_1^2 \int_0^1 (1 - 4\xi + 4\xi^2)d\xi = \frac{4}{\beta}A_1^2(1 - 2 + \frac{4}{3}),$$

$$(C_d)_{\alpha=0} = \frac{4}{3}\frac{A_1^2}{\beta}.$$

and:

$$(C_{m,o})_{\alpha=0} = \frac{4}{\beta}\int_0^c d^{'}(x)\frac{x}{c}\frac{dx}{c} = \frac{4}{\beta}A_1 \int_0^1 (\xi - 2\xi^2)d\xi = \frac{4}{\beta}A_1(\frac{1}{2} - \frac{2}{3}),$$

$$(C_{m,o})_{\alpha=0} = -\frac{2}{3}\frac{A_1}{\beta}.$$

2.2 Global Coefficient at $\alpha \neq 0$

For arbitrary α, the coefficients are:

$$C_l = \frac{4\alpha}{\beta},\ C_d = \frac{4}{3}\frac{A_1^2}{\beta} + \frac{4\alpha^2}{\beta},\ C_{m,o} = -\frac{2}{3}\frac{A_1}{\beta} - \frac{2\alpha}{\beta}.$$

If thickness were added to the cambered plate, of these coefficients, C_l, C_d and $C_{m,o}$, only C_d would be affected.

The maximum “finesse”, for $A_1 = 0.1$, is obtained when $(C_d)_{fx} = 2(C_d)_{\alpha=0}$ (property of the parabola), i.e. for $\alpha_{fx} = \frac{A_1}{\sqrt{3}}$. Hence, $(C_l)_{fx} = \frac{4}{\sqrt{3}}\frac{A_1}{\beta}$ resulting in:

$$f_x = \frac{\sqrt{3}}{2}\frac{1}{A_1} = 8.66$$