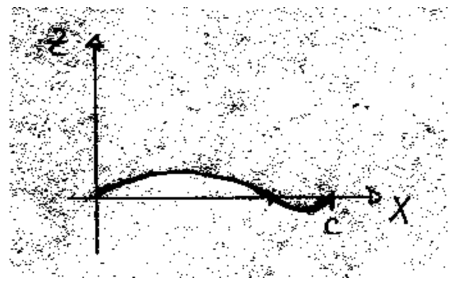
EAE 127 – MIDTERM 11/08/2005

SOLUTION

(Open notes, open book)

1. Inviscid, Incompressible Flow past a Thin Airfoil (15 points)

1.1 Sketch the camber line in the (x,z)-plane.



1.2 Expanding *d(x)* yields

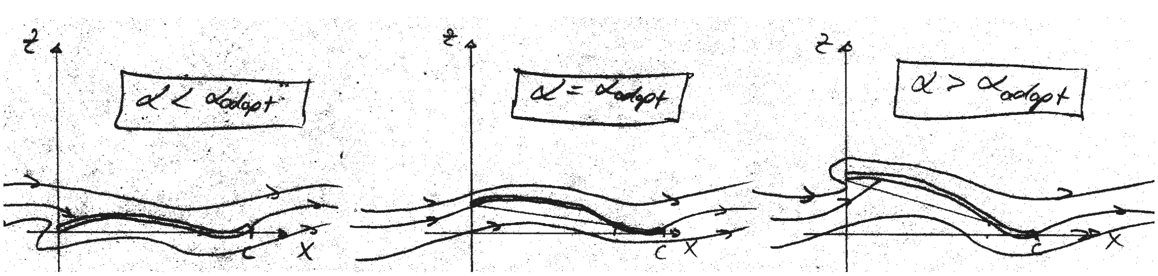
$$d(x)=\frac{1}{3}Ac\left(3\frac{x}{c}-7\frac{x^2}{c^2}+4\frac{x^3}{c^3}\right) \quad \text{and} \quad d'(x)=\frac{1}{3}A\left(3-14\frac{x}{c}+12\frac{x^2}{c^2}\right).$$

Using the transformation $\frac{x}{c}=\frac{1}{2}[1-\cos(t)]$ and the identity $\cos^2 t=\frac{1}{2}[1+\cos(2t)]$ …

$$d'[x(t)]=\frac{A}{6}+\frac{A}{3}\cos(t)+\frac{A}{2}\cos(2t)$$

Thus, $-A_0+\sum_{n=1}^{\infty}A_n\cdot\cos(nt)=d'[x(t)]-a=\frac{A}{6}-a+\frac{A}{3}\cos(t)+\frac{A}{2}\cos(2t)$ and the coefficients A_n are found to be … $\boxed{A_0=a-\frac{A}{6}}$, $\boxed{A_1=\frac{A}{3}}$, $\boxed{A_2=\frac{A}{2}}$, $\boxed{A_3=...=A_n=0}$.

1.3 At the angle of adaptation a_{adapt}, $\quad A_0=0=a-a_{adapt} \quad$ => $\quad \boxed{a_{adapt}=\frac{A}{6}}$



1.4 The aerodynamic coefficients are

$$c_l = 2p\left(A_0 + \frac{A_1}{2}\right) \qquad => \qquad \boxed{c_l = 2pa}$$

$$\boxed{c_d = 0}$$

$$c_{m,0} = -\frac{p}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right) \qquad => \qquad \boxed{c_{m,0} = -\frac{p}{2}\left(a - \frac{A}{12}\right)}$$

1.5 The aerodynamic center $\frac{x_{a.c.}}{c}$ and the center of pressure $\frac{x_{c.p.}}{c}$ are

$$\boxed{\frac{x_{a.c.}}{c} = \frac{1}{4}} \qquad \text{for ‘all’ thin airfoils}$$

$$\frac{x_{c.p.}}{c} = -\frac{c_{m,0}}{c_l} \qquad => \qquad \boxed{\frac{x_{c.p.}}{c} = \frac{1}{4}\frac{a - A/12}{a}}$$

1.6 The angle of attack a for which $\frac{x_{c.p.}}{c} = \frac{1}{2}$

$$\frac{1}{2} = \frac{x_{c.p.}}{c} = \frac{1}{4}\frac{a - A/12}{a} \qquad => \qquad \boxed{a = -\frac{A}{12}}$$

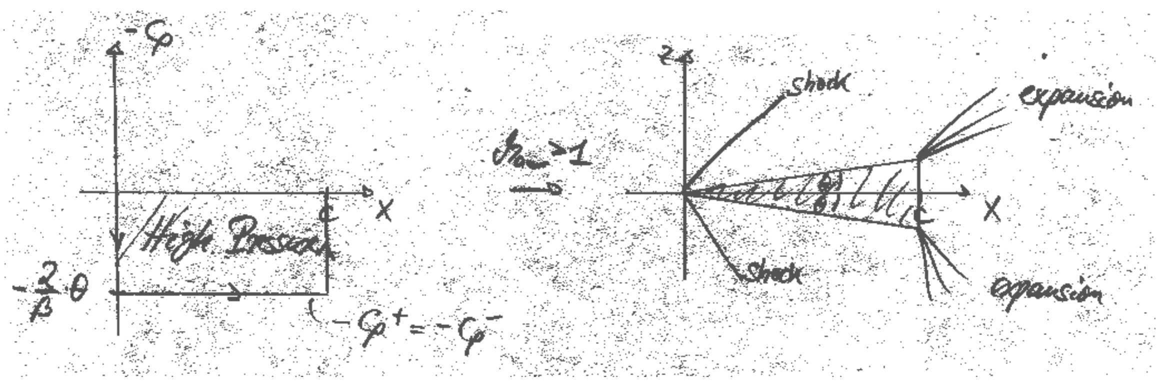
2. Linearized Supersonic Flow past a Thin Airfoil (15 points)

$$f'^{+}(x) = \frac{e'(x)}{2} = \Theta \qquad\qquad f'^{-}(x) = -\frac{e'(x)}{2} = -\Theta$$

2.1 At $a = 0$, …

Ackeret

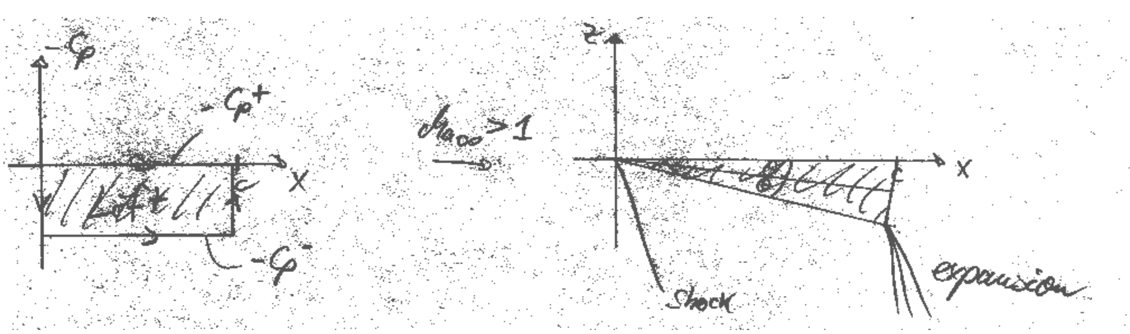
$$\boxed{\begin{aligned} c_p^{+}(x) &= \frac{2}{b}\left(f'^{+}(x) - a\right) = \frac{2}{b}\Theta \\ c_p^{-}(x) &= -\frac{2}{b}\left(f'^{-}(x) - a\right) = \frac{2}{b}\Theta \end{aligned}}$$



2.2 At $a = \Theta$, …

Ackeret

$$\boxed{\begin{aligned} c_p^{+}(x) &= \frac{2}{b}\left(f'^{+}(x) - a\right) = \frac{2}{b}(\Theta - \Theta) = 0 \\ c_p^{-}(x) &= -\frac{2}{b}\left(f'^{-}(x) - a\right) = -\frac{2}{b}(-\Theta - \Theta) = \frac{4}{b}\Theta \end{aligned}}$$



2.3 The aerodynamic coefficients are

$$\boxed{c_l = \frac{4a}{b}}$$

$$c_d = (c_d)_{a=0} + \frac{4a^2}{b} \quad => \quad (c_d)_{a=0} = \frac{4}{b}\int_0^c \left(\frac{e'(x)}{2}\right)^2 \frac{dx}{c} = \frac{4}{b}\int_0^c \Theta^2 \frac{dx}{c} = \frac{4\Theta^2}{b}$$

$$\boxed{c_d = \frac{4}{b}\left(\Theta^2 + a^2\right)}$$

$$c_{m,0} = \left(c_{m,0}\right)_{a=0} - \frac{2a}{b} \quad => \quad \left(c_{m,0}\right)_{a=0} = \frac{4}{b}\int_0^c d'(x)\frac{x}{c}\frac{dx}{c} = 0$$

$$\boxed{c_{m,0} = -\frac{2a}{b}}$$

2.4 The aerodynamic center $\frac{x_{a.c.}}{c}$ and the center of pressure $\frac{x_{c.p.}}{c}$ are

$$\boxed{\frac{x_{a.c.}}{c} = \frac{1}{2}} \qquad \text{for 'all' thin airfoils}$$

$$\frac{x_{c.p.}}{c} = \frac{1}{2} - b\,\frac{\left(c_{m,0}\right)_{a=0}}{4a} \quad => \quad \boxed{\frac{x_{c.p.}}{c} = \frac{1}{2}}$$

2.5 Why are supersonic airfoils usually symmetric, i.e. without camber ?

Because camber does not produce lift but only contributes to the drag.

2.6 Maximum Finess

$$c_l = \sqrt{\frac{4}{b}(c_d)_{a=0}} \quad => \quad \boxed{c_l = \frac{4\Theta}{b}}$$

$$c_d = 2(c_d)_{a=0} \quad => \quad \boxed{c_d = \frac{8\Theta^2}{b}}$$

$$c_l = \frac{4a}{b} = \frac{4\Theta}{b} \quad => \quad \boxed{a = \Theta}$$