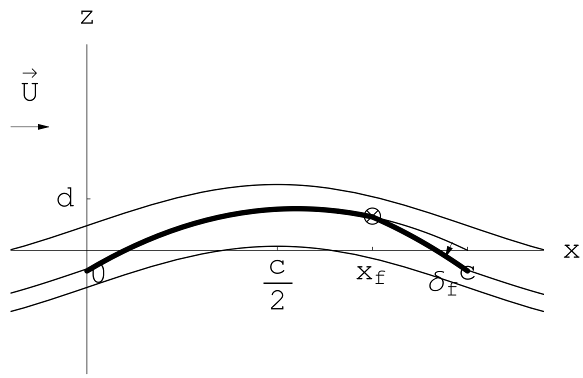

Figure 1: Sketch of flow at α_{adapt}

EAE 127 - MIDTERM/SOLUTION 11/03/06

(Open Notes, open Book)

1. Inviscid, Incompressible Flow (20 points)

1.1 Flap Modelization with Thin Airfoil Theory

The slope $d'(x)$ of the plate as a function of x is given by

$$\begin{cases} d'(x) = 4\frac{d}{c}(1-2\frac{x}{c}),\ 0 \le x \le x_f \\ d'(x) = 4\frac{d}{c}(1-2\frac{x}{c}) - \delta_f,\ x_f \le x \le c \end{cases}$$

As a function of t one gets

$$\begin{cases} d'(x) = 4\frac{d}{c}\cos t,\ 0 \le t \le t_f \\ d'(x) = 4\frac{d}{c}\cos t - \delta_f,\ t_f \le xt \le \pi \end{cases}$$

1.2 Coefficient A_0

Using the formula derived in class

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi d'[x(t)]dt = \alpha - \frac{1}{\pi}\int_0^\pi 4\frac{d}{c}\cos t dt + \frac{\delta_f}{\pi}\int_{t_f}^\pi dt = \alpha + (1-\frac{t_f}{\pi})\delta_f$$

For $t_f = \pi$ ($x_f = c$) one finds $A_0 = \alpha$. In this case there is no flap and the parabolic plate result is recovered.
For $t_f = 0$ ($x_f = 0$) one finds $A_0 = \alpha + \delta_f$. The whole plate is a flap, therefore the flap angle has the same effect as the angle of incidence.
The angle of adaptation $\alpha_{adapt}(t_f, \delta_f)$ is

$$\alpha_{adapt}(t_f, \delta_f) = -(1-\frac{t_f}{\pi})\delta_f$$

For $t_f = \pi$ ($x_f = c$) $\alpha_{adapt} = 0$. This is the result for the parabolic plate.
For $t_f = 0$ ($x_f = 0$) $\alpha_{adapt} = -\delta_f$. The incidence and the flap deflection are opposite, so that the profile chord is parallel to the incoming flow.
The incidence of adaptation for $x_f = 3c/4$ ($t_f = 2\pi/3$) and $\delta_f = 10\ deg$ is $\alpha_{adapt} = -\frac{1}{3}\delta_f = -3.3\ deg$.
See Fig. 1.

1.3 Coefficient A_1

A_1 is given by

$$A_1 = \frac{2}{\pi}\int_0^\pi d'[x(t)]\cos t dt = \frac{2}{\pi}\int_0^\pi 4\frac{d}{c}\cos^2 t dt - \frac{2\delta_f}{\pi}\int_{t_f}^\pi \cos t dt = 4\frac{d}{c} + \frac{2}{\pi}\sin t_f\,\delta_f$$

For $t_f = \pi$ ($x_f = c$) the parabolic plate result $A_1 = \frac{4d}{c}$ is recovered.
For $t_f = 0$ ($x_f = 0$) the profile remains a parabolic thin profile, resulting in the same value $A_1 = \frac{4d}{c}$.

1.4 Coefficients $A_2, ..., A_n,\ n \geq 2$

The general term A_n is given by

$$A_n = \frac{2}{\pi}\int_0^\pi d'[x(t)]\cos nt dt = \frac{2}{\pi}\int_0^\pi 4\frac{d}{c}\cos t\cos nt dt - \frac{2\delta_f}{\pi}\int_{t_f}^\pi \cos nt dt = \frac{\sin nt_f}{n}\delta_f$$

For $n = 2$ this becomes $A_2 = \frac{\sin 2t_f}{2}\delta_f$.

1.5 Global Coefficients

As shown in class $C_l = 2\pi(A_0 + A_1/2)$. Substituting the expressions for the Fourier coefficients

$$C_l = 2\pi\{\alpha + 2\frac{d}{c} + [1 - \frac{t_f}{\pi} + \frac{\sin t_f}{\pi}]\delta_f\}$$

The leading edge pitching moment is $C_{m,o} = -\frac{\pi}{2}(A_0 + A_1 - A_2/2)$. This becomes

$$C_{m,o} = -\frac{\pi}{2}\{\alpha + 4\frac{d}{c} + [1 - \frac{t_f}{\pi} + 2\frac{\sin t_f}{\pi} - \frac{\sin 2t_f}{4}]\delta_f\}$$

The drag coefficient is always zero

$$C_d = 0$$

If thickness is added to the thin cambered plate, none of these coefficients will be affected.

1.6 Aerodynamic Center (Bonus Points)

The *aerodynamic center* is the point about which the moment of the aerodynamic forces is independent of incidence.
For a thin airfoil it is at quarter-chord.
The moment at the aerodynamic center is given by $C_{m,ac} = -\frac{\pi}{4}(A_1 - A_2)$, that is

$$C_{m,ac} = -\frac{\pi}{4}\{4\frac{d}{c} + [\frac{2\sin t_f}{\pi} - \frac{\sin 2t_f}{2}]\delta_f\}$$

2. Linearized Supersonic Flow (10 points)

2.1 Flap Modelization with Thin Airfoil Theory

The slope of the mean camberline is given by

$$\begin{cases} d'(x) = 0,\ 0 \leq x \leq x_f \\ d'(x) = -\delta_f,\ x_f \leq x \leq c \end{cases}$$

As seen in class, the Ackret formulae give the pressure coefficients as

$$\begin{cases} C_p^+(x) = \frac{2}{\beta}(d'(x) - \alpha) = -\frac{2}{\beta}\alpha,\ 0 \leq x \leq x_f \\ C_p^+(x) = \frac{2}{\beta}(d'(x) - \alpha) = -\frac{2}{\beta}(\alpha + \delta_f),\ x_f \leq x \leq c \end{cases}$$

$$\begin{cases} C_p^-(x) = -\frac{2}{\beta}(d'(x) - \alpha) = \frac{2}{\beta}\alpha,\ 0 \leq x \leq x_f \\ C_p^-(x) = -\frac{2}{\beta}(d'(x) - \alpha) = \frac{2}{\beta}(\alpha + \delta_f),\ x_f \leq x \leq c \end{cases}$$

See Fig. 2.

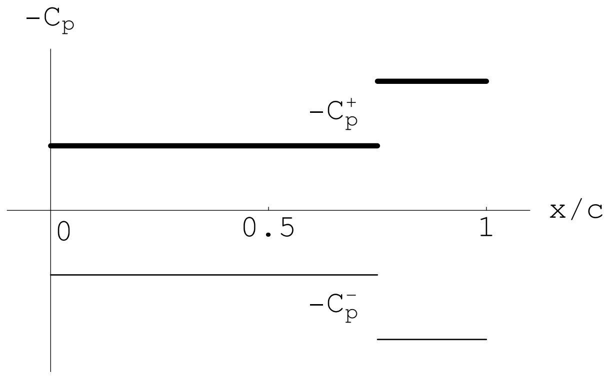

Figure 2: Pressure coefficient distributions on the flat plate with flap

2.1 Global Coefficients: C_l

The aerodynamic coefficient C_l is obtained by integration of the pressure coefficients according to

$$C_l = \int_0^c (C_p^-(x) - C_p^+(x))\frac{dx}{c} = \int_{x_f}^c 4\frac{\delta_f}{\beta}\frac{dx}{c} + \int_0^c 4\frac{\alpha}{\beta}\frac{dx}{c} = 4\frac{1-\frac{x_f}{c}}{\beta}\delta_f + 4\frac{\alpha}{\beta}$$

2.2 Global Coefficient: C_d

$C_d = \frac{4}{\beta}\int_0^c (d'^2 + e'^2/4 + \alpha^2)\frac{dx}{c}$. The second term vanishes (zero thickness). There remains

$$C_d = \frac{4}{\beta}\int_{x_f}^c \delta_f^2 \frac{dx}{c} + \frac{4}{\beta}\int_0^c \alpha^2 \frac{dx}{c} = 4\frac{1-\frac{x_f}{c}}{\beta}\delta_f^2 + 4\frac{\alpha^2}{\beta}$$