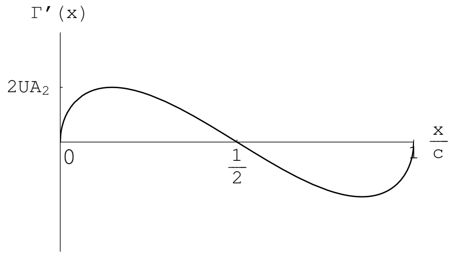

Figure 1: Vorticity distribution for mode 2

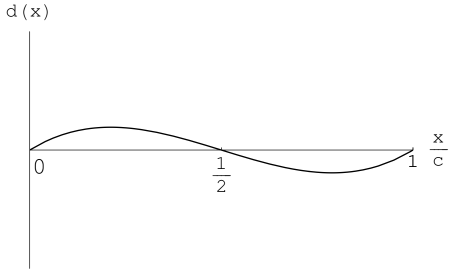

Figure 2: Camberline shape

EAE 127 - MIDTERM/SOLUTION 11/02/07

1. Inviscid, Incompressible Flow (20 points)

1.1 Second Mode Airfoil Shape

$$\begin{cases} \Gamma'[x(t)] = 2UA_2 \sin 2t \\ x(t) = \frac{c}{2}(1-\cos t),\ 0 \le t \le \pi \end{cases}$$

The sketch of $\Gamma'(x)$ is shown in Fig. 1.
The camberline slope is given by $d'[x(t)] = \alpha - A_0 + \sum_{n=1}^{\infty} A_n \cos nt = \alpha - A_0 + A_2 \cos 2t$, in our case. Given the identity $\cos 2t = 2\cos^2 t - 1 = 2\left(1-2\frac{x}{c}\right)^2 - 1 = 1 - 8\frac{x}{c} + 8(\frac{x}{c})^2$, one finds

$$d'(x) = \alpha - A_0 + A_2\left(1 - 8\frac{x}{c} + 8(\frac{x}{c})^2\right)$$

Integrating from zero to x

$$d(x) = (\alpha - A_0 + A_2)x - 8A_2\frac{x^2}{2c} + 8A_2\frac{x^3}{3c^2}$$

The condition $d(0) = 0$ is automatically satisfied, but the condition $d(c) = 0$ provides an equation that defines A_0 as

$$d(c) = \left(\alpha - A_0 + A_2 - 4A_2 + 8\frac{A_2}{3}\right)c = 0, \quad \Rightarrow A_0 = \alpha - \frac{A_2}{3}$$

Substitution of A_0 in $d(x)$, and factoring out the roots of this 3rd degree polynomial yields

$$d(x) = 4\frac{A_2 c}{3}\,\frac{x}{c}\left(1-2\frac{x}{c}\right)\left(1-\frac{x}{c}\right)$$

See Fig. 2.

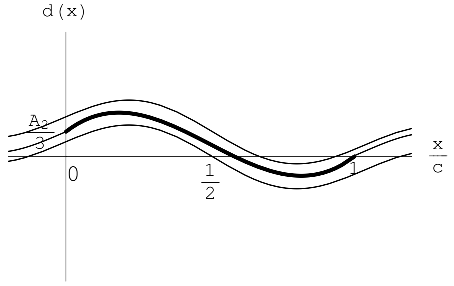

Figure 3: Flow solution at incidence of adaptation

1.2 Second Mode Aerodynamic Characteristics

The *incidence of adaptation* (ideal angle of attack) is such that $A_0 = 0$. In our case, this corresponds to $\alpha = A_2/3$. At the incidence of adaptation, the solution satisfies two Kutta-Joukowski conditions, at the trailing edge, but also at the leading edge. See Fig. 3.
The lift coefficient is $C_l(\alpha) = 2\pi(A_0 + A_1/2) = 2\pi(\alpha - A_2/3)$.
At incidence of adaptation, the lift coefficient is $C_l = 0$. At $\alpha = 0$ the lift coefficient is $C_l = -2\pi A_2/3$.
The moment coefficient is $C_{m,o}(\alpha) = -\pi(A_0 + A_1 - A_2/2)/2 = -\pi(\alpha - 5A_2/6)/2$. Using the change of moment formula, the moment $C_{m,\frac{c}{2}}(\alpha)$ at the mid-chord reads

$$C_{m,\frac{c}{2}}(\alpha) = C_{m,o}(\alpha) + \frac{1}{2}C_l(\alpha) = \frac{\pi}{2}\left(\alpha + \frac{A_2}{6}\right)$$

1.3 Static Stability About an Axis

The incidence of equilibrium, α_{eq}, about an axis located at mid-chord corresponds to $C_{m,\frac{c}{2}}(\alpha_{eq}) = 0$. This is verified for $\alpha_{eq} = -A_2/6$.
Study of static stability of the profile at $\alpha = \alpha_{eq}$ is done by perturbing the equilibrium with, say, a $\delta\alpha > 0$. In this case, the flow develops a $\delta C_{m,\frac{c}{2}} > 0$, which is a nose up moment. Hence the incidence will increase away from equilibrium. The static equilibrium is unstable. Another proof is by noticing that the sign of the moment slope is positive:

$$\frac{dC_{m,\frac{c}{2}}(\alpha)}{d\alpha} = \frac{\pi}{2} > 0$$

2. Linearized Supersonic Flow (10 points)

2.1 Pressure Distribution on a Thin Cambered Plate

Let $\beta = \sqrt{M_\infty^2 - 1}$, $M_\infty > 1$. Consider a thin cambered plate of equation:

$$d(x) = Ac\frac{x}{c}\left(1 - 2\frac{x}{c}\right)\left(1 - \frac{x}{c}\right)$$

the slope $d'(x)$ is given by

$$d'(x) = A\left(1 - 6\frac{x}{c} + 6\frac{x^2}{c^2}\right)$$

The pressure coefficients $C_p^+(x)$ and $C_p^-(x)$ are

$$\left\{\begin{array}{l} C_p^+(x) = \frac{2A}{\beta}(1 - 6\frac{x}{c} + 6\frac{x^2}{c^2}) \\ C_p^-(x) = -\frac{2A}{\beta}(1 - 6\frac{x}{c} + 6\frac{x^2}{c^2}) \end{array}\right.$$

See Fig. 4.

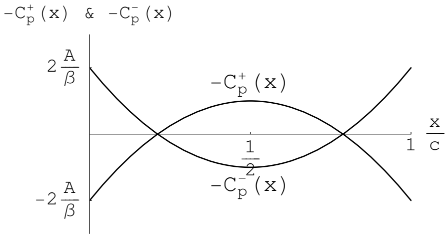

Figure 4: Pressure coefficients distributions at $\alpha = 0$

2.2 Global Coefficients: C_l, C_d, $C_{m,o}$

As seen in class, $C_l(\alpha) = 4\alpha/\beta$. The aerodynamic coefficients $(C_d)_{\alpha=0}$, is given by the integral

$$(C_d)_{\alpha=0} = \frac{4}{\beta}\int_0^c d'^2(x)\frac{dx}{c} = \frac{4A^2}{\beta}\int_0^1 (1-6\xi+6\xi^2)^2 d\xi = \frac{4A^2}{\beta}\int_0^1 (1-12\xi+48\xi^2-72\xi^3+36\xi^4)d\xi$$

The results is

$$(C_d)_{\alpha=0} = \frac{4A^2}{5\beta}$$

Similarly, $(C_{m,o})_{\alpha=0}$, is given by

$$(C_{m,o})_{\alpha=0} = \frac{4}{\beta}\int_0^c d'(x)\frac{x}{c}\frac{dx}{c} = \frac{4A}{\beta}\int_0^1 (1-6\xi+6\xi^2)\xi d\xi = \frac{4A^2}{\beta}\int_0^1 (\xi-6\xi^2+6\xi^3)d\xi$$

resulting in

$$(C_{m,o})_{\alpha=0} = 0$$

2.3 Static Stability About an Axis

The moment at mid-chord is $C_{m,\frac{c}{2}}(\alpha) = C_{m,o}(\alpha) + C_l(\alpha)/2$, using the change of moment formula. One finds

$$C_{m,\frac{c}{2}}(\alpha) = -2\frac{\alpha}{\beta} + \frac{1}{2}4\alpha = 0$$

The equilibrium is neutrally stable.