EAE 127 - MIDTERM/SOLUTION 11/05/08

1. Inviscid, Incompressible Flow (20 points)

1.1 Definition of the Center of Pressure

The Center of Pressure is the point about which the moment of the aerodynamic forces is zero.

1.2 Finding the Fourier Coefficients

When the leading edge is "adapted" the coefficient of the singular term in the expansion of the vorticity is zero, i.e. $A_0 = 0$.
The design requirement is

$$\frac{x_{c.p.}}{c} = -\frac{C_{m,0}}{C_l} = \frac{A_0 + A_1 - \frac{A_2}{2}}{4(A_0 + \frac{A_1}{2})} = \frac{1}{2}$$

Since the denominator is not zero, upon multiplying by it one gets

$$A_0 + A_1 - \frac{A_2}{2} = 2(A_0 + \frac{A_1}{2})$$

Note that A_1 drops out and is undetermined at this point. Since the flow is "adapted", $A_0 = 0$. This requirement results in $A_2 = 0$. The "simplest profile" thus has $A_2 = A_3 = ... = A_n = 0$, $n \geq 2$. The coefficient A_0 will depend on α and A_1 remains to be determined as well. As seen in class, the profile represented by the first regular term in the Fourier series is a parabolic cambered plate. This results from

$$d'[x(t)] = \alpha - A_0 + \sum_{n=1}^{\infty} A_n \cos nt = \alpha - A_0 + A_1 \cos t = \alpha - A_0 + A_1 \left(1 - 2\frac{x}{c}\right)$$

Upon integration one obtains

$$d(x) = \int_0^x \left(\alpha - A_0 + A_1 \left(1 - 2\frac{\xi}{c}\right)\right) d\xi = (\alpha - A_0 + A_1)\, x - A_1 \frac{x^2}{c}$$

Enforcing the condition that $d(c) = 0$ determines A_0 as

$$(\alpha - A_0 + A_1)\, c - A_1 c = 0, \quad \Rightarrow \quad A_0 = \alpha$$

Finally, the lift coefficient is given by

$$C_l = 2\pi \left(A_0 + \frac{A_1}{2}\right) = 2\pi \left(\alpha + \frac{A_1}{2}\right) = \pi A_1 = C_{l,adapt} = \frac{1}{2}$$

This results in

$$A_1 = \frac{1}{2\pi}$$

1.3 Equation of the Cambered Plate

From the above results, the equation of the cambered plate is

$$d(x) = A_1 c \frac{x}{c}\left(1 - \frac{x}{c}\right) = \frac{c}{2\pi}\frac{x}{c}\left(1 - \frac{x}{c}\right)$$

As seen in class, $A_1 = 4\frac{d}{c}$, hence the relative camber of the parabolic profile is

$$\frac{d}{c} = \frac{1}{8\pi} = 0.04$$

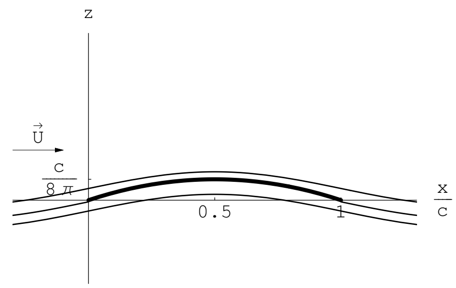

Figure 1: Flow past thin cambered parabolic plate at adaptation incidence $\alpha = 0$

1.4 Results

The incidence of adaptation is $\alpha = 0$.
The sketch of the flow is shown in Fig. 1.
At adaptation, the lift coefficient is by design

$$C_{l,adapt} = \frac{1}{2}$$

The moment coefficient is

$$C_{m,0} = -\frac{\pi}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right) = -\frac{1}{4}$$

The drag coefficient is always

$$C_d = 0$$

2. Linearized Supersonic Flow (20 points)

Let $\beta = \sqrt{M_\infty^2 - 1}$, $M_\infty > 1$. Consider a thin parabolic plate of equation:

$$d(x) = 4d\,\frac{x}{c}\left(1 - \frac{x}{c}\right), \quad d > 0$$

2.1 Pressure Distribution on a Thin Cambered Plate

the slope $d'(x)$ is given by

$$d'(x) = 4\frac{d}{c}\left(1 - 2\frac{x}{c}\right)$$

The pressure coefficients $C_p^+(x)$ and $C_p^-(x)$ are

$$\begin{cases} C_p^+(x) = \frac{8}{\beta}\frac{d}{c}(1 - 2\frac{x}{c}) \\ C_p^-(x) = -\frac{8}{\beta}\frac{d}{c}(1 - 2\frac{x}{c}) \end{cases}$$

See Fig. 2.

2.2 Global Coefficients: C_l, C_d, $C_{m,o}$

As seen in class, $C_l(\alpha) = 4\alpha/\beta$. The aerodynamic coefficients $(C_d)_{\alpha=0}$, is given by the integral

$$(C_d)_{\alpha=0} = \frac{4}{\beta}\int_0^c d'^2(x)\frac{dx}{c} = \frac{64}{\beta}\left(\frac{d}{c}\right)^2 \int_0^1 (1 - 4\xi + 4\xi^2)^2 d\xi = \frac{64}{\beta}\left(\frac{d}{c}\right)^2 (1 - 2 + \frac{4}{3})$$

The results is

$$(C_d)_{\alpha=0} = \frac{64}{3\beta}\left(\frac{d}{c}\right)^2$$

Similarly, $(C_{m,o})_{\alpha=0}$, is given by

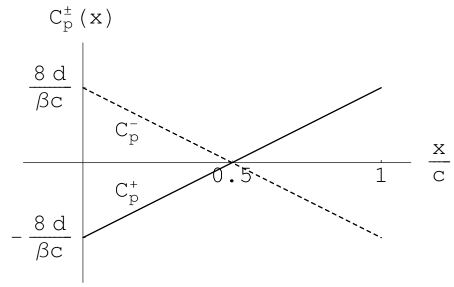

Figure 2: Pressure coefficients distributions at $\alpha = 0$

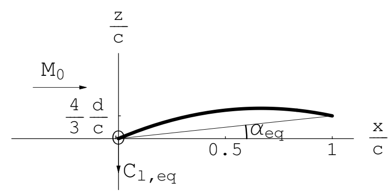

Figure 3: Parabolic cambered plate in equilibrium about leading edge axis at α_{eq}

$$(C_{m,o})_{\alpha=0} = \frac{4}{\beta}\int_0^c d'(x)\frac{x}{c}\frac{dx}{c} = \frac{16}{\beta}\frac{d}{c}\int_0^1 (1-2\xi)\xi d\xi = \frac{16}{\beta}\frac{d}{c}(\frac{1}{2}-\frac{2}{3})$$

resulting in

$$(C_{m,o})_{\alpha=0} = -\frac{8}{3\beta}\frac{d}{c}$$

2.3 Static Equilibrium About an Axis

The moment at the leading edge is $C_{m,o}(\alpha) = (C_{m,o})_{\alpha=0} - 2\frac{\alpha}{\beta}$. Equilibrium corresponds to $C_{m,o}(\alpha_{eq}) = 0$, i.e.

$$\alpha_{eq} = \frac{\beta}{2}(C_{m,o})_{\alpha=0} = -\frac{4}{3}\frac{d}{c}$$

The corresponding value of the lift coefficient is

$$C_{l,eq} = -\frac{16}{3\beta}\frac{d}{c}$$

See Fig. 3.

2.4 Static Stability

The equilibrium is stable since $\frac{dC_{m,o}}{d\alpha} = -\frac{2}{\beta} < 0$.