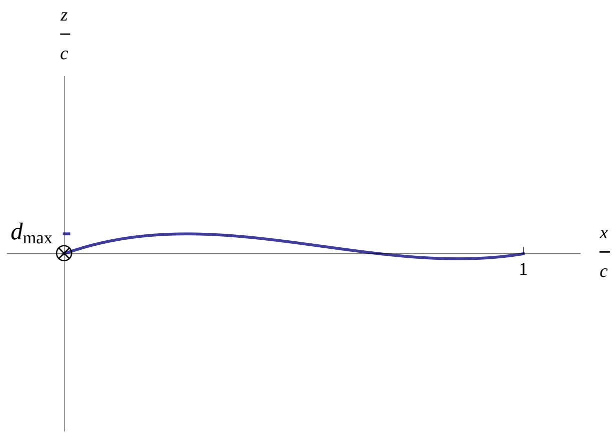

Figure 1: Thin cambered plate geometry

EAE 127 - MIDTERM/SOLUTION 11/02/09

(Open Notes, open Book)

1. Inviscid, Incompressible Flow (20 points)

Design of a Thin Cambered Plate: We wish to design the *simplest* cambered plate with a hinge located at the leading edge. The hinge is clamped and the thin airfoil will rotate without friction to an equilibrium position corresponding to a specified lift coefficient $C_{l,eq}$ such that the leading edge is "adapted". We will neglect the weight of the plate. By "simplest" plate we mean a plate that has the smallest possible number of non-zero Fourier coefficients $A_1, A_2, ...A_n$ for Γ'.

1.1 Definition of the Center of Pressure

The *Center of Pressure* is the point about which the moment of the aerodynamic forces is zero.

1.2 Condition for Equilibrium

The condition for equilibrium reads $C_{m,o} = -\frac{\pi}{2}(A_0 + A_1 - A_2/2) = 0$.
The condition on the Fourier coefficients to satisfy this equilibrium requirement is $A_0 + A_1 - A_2/2 = 0$.

1.3 Leading Edge "Adaptation"

Leading edge "adaptation" corresponds to that unique value of incidence for which the profile statisfies two Kutta-Joukowski conditions, one at the trailing edge (always) and one at the leading edge (exceptionally).
The singular term in the Fourier series expansion of Γ' is zero, i.e. $A_0 = 0$.
Accounting for adapted leading edge, the coefficients for the simplest plate read: A_0 arbitrary, A_1 to be determined, $A_2 = 2A_1$, $A_3 = A_4 = ... = A_n = 0$, $n \geq 3$.

1.4 Lift Coefficient at Equilibrium

The lift coefficient is given by $C_l = 2\pi(A_0 + A_1/2)$.
The Fourier coefficients needed to satisfy the constraint $C_l = C_{l,eq}$ are given by

$$A_1 = \frac{C_{l,eq}}{\pi}, \quad A_2 = 2\frac{C_{l,eq}}{\pi}$$

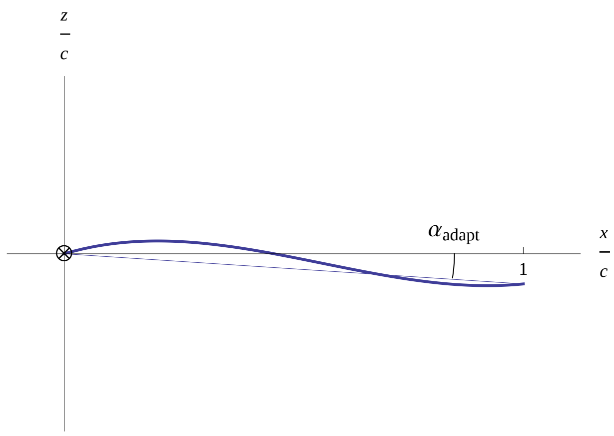

Figure 2: Thin cambered plate at equilibrium $M = 0$

1.5 Equation of the Cambered Plate

The equation of the cambered plate is obtained from

$$d'[x(t)] = \alpha - A_0 + \sum_{n=1}^{\infty} A_n \cos nt = \alpha - A_0 + A_1 \cos t + 2A_1 \cos 2t$$

Upon elimination of the parameter t for x, one gets

$$d'(x) = \alpha - A_0 + A_1 \left(1 - 2\frac{x}{c}\right) + 2A_1 \left(1 - \frac{8x}{c} + \frac{8x^2}{c^2}\right) = \alpha - A_0 + 3A_1 - 18A_1\frac{x}{c} + 16A_1\frac{x^2}{c^2}$$

Integrating from zero to x gives

$$d(x) = (\alpha - A_0 + 3A_1)\, x - 9A_1\frac{x^2}{c} + \frac{16}{3}A_1\frac{x^3}{c^2}$$

We have two boundary conditions: $d(0) = d(c) = 0$. The first one is accounted for by the integration from $x = 0$. The second one will determine A_0.

$$d(c) = c\left(\alpha - A_0 + 3A_1 - 9A_1 + \frac{16}{3}A_1\right) = 0$$

This gives

$$A_0 = \alpha - \frac{2}{3}A_1$$

The equation of the plate can be written as a product of first degree terms (see Fig. 1)

$$d(x) = \frac{A_1 c}{3}\frac{x}{c}\left(11 - 16\frac{x}{c}\right)\left(1 - \frac{x}{c}\right) = \frac{C_{l,eq}c}{3\pi}\frac{x}{c}\left(11 - 16\frac{x}{c}\right)\left(1 - \frac{x}{c}\right)$$

1.6 Plate Aerodynamic Coefficients

The aerodynamic coefficients, C_l, $C_{m,o}$ and C_d, will now read

$$C_l(\alpha) = 2\pi\left(\alpha - \frac{C_{l,eq}}{6\pi}\right), \quad C_{m,o}(\alpha) = -\frac{\pi}{2}\left(\alpha - \frac{2C_{l,eq}}{3\pi}\right), \quad C_d = 0$$

The angle of adaptation or *ideal angle of attack* for this plate corresponds to $A_0 = 0$, hence

$$\alpha_{adapt} = \frac{2C_{l,eq}}{3\pi}$$

Checking the design conditions:

$$C_l(\alpha_{adapt}) = C_{l,eq}, \quad C_{m,o}(\alpha_{adapt}) = 0$$

which were the design requirements (see Fig. 2).

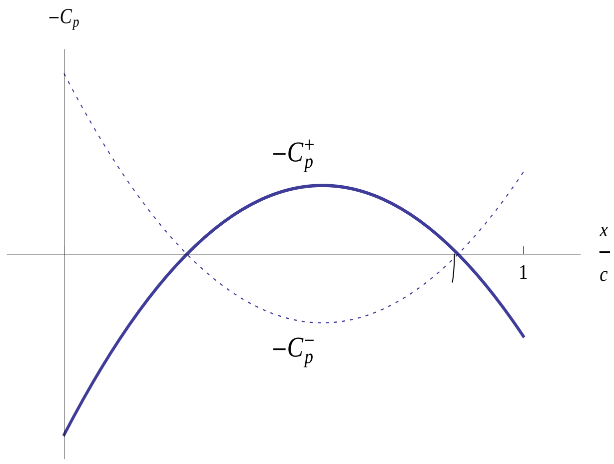

Figure 3: Pressure coefficients distributions at $\alpha = 0$

2. Linearized Supersonic Flow (10 points)

Let $\beta = \sqrt{M_\infty^2 - 1}$, $M_\infty > 1$. Consider a thin cambered plate of equation

$$d(x) = d\left(11\frac{x}{c} - 27\frac{x^2}{c^2} + 16\frac{x^3}{c^3}\right)$$

d is related to the camber ($d > 0$).

2.1 Pressure Distribution on the Thin Cambered Plate

The pressure coefficients $C_p^+(x)$ and $C_p^-(x)$ along the plate, for $\alpha = 0$ are given by (see Fig. 3)

$$C_p^+ = \frac{2}{\beta}\frac{d}{c}\left(11 - 54\frac{x}{c} + 48\frac{x^2}{c^2}\right)$$

$$C_p^- = -\frac{2}{\beta}\frac{d}{c}\left(11 - 54\frac{x}{c} + 48\frac{x^2}{c^2}\right)$$

2.2 Global Coefficients: C_l, $C_{m,o}$

The lift coefficient is $C_l(\alpha) = 4\alpha/\beta$.
The aerodynamic coefficient $(C_{m,o})_{\alpha=0}$ is obtained from

$$(C_{m,o})_{\alpha=0} = \frac{4}{\beta}\int_0^c d'(x)\frac{x}{c}\frac{dx}{c} = \frac{4}{\beta}\frac{d}{c}\int_0^c \left(11 - 54\xi + 48\xi^2\right)\xi d\xi = \frac{4}{\beta}\frac{d}{c}\left(\frac{11}{2} - \frac{54}{3} + 12\right) = -\frac{2}{\beta}\frac{d}{c}$$

The aerodynamic moment coefficient is

$$C_{m,o}(\alpha) = -\frac{2}{\beta}\frac{d}{c} - \frac{2\alpha}{\beta}$$

2.3 Static Equilibrium About an Axis

The equilibrium angle α_{eq} corresponds to $C_{m,o}(\alpha_{eq}) = 0$, hence

$$\alpha_{eq} = -\frac{d}{c}$$

and $C_{l,eq}$ is

$$C_{l,eq} = \frac{4\alpha_{eq}}{\beta} = -\frac{4}{\beta}\frac{d}{c}$$

The sketch of the profile at equilibrium with vector $\overrightarrow{C}_{l,eq}$ is shown in Fig. 4.

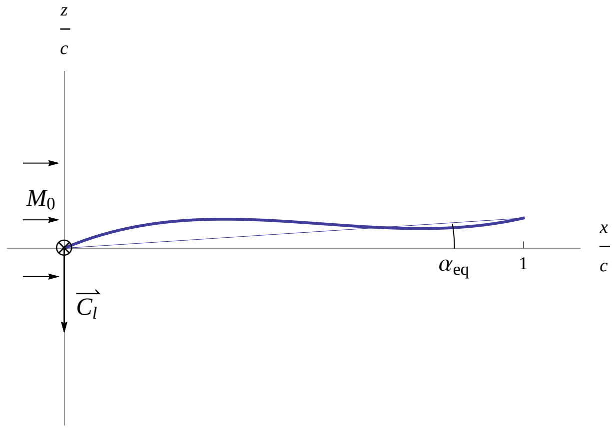

Figure 4: Free body diagram at equilibrium $M_0 > 1$

2.4 Static Stability

The equilibrium is statically stable since

$$\frac{dC_{m,o}(\alpha)}{d\alpha} = -\frac{2}{\beta} < 0$$