

Aerodynamics of the Curve-Ball: An Investigation of the  
Effects of Angular Velocity on Baseball Trajectories

By

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To Dad

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## ABBREVIATIONS

The following is a list of abbreviations and symbols used throughout this dissertation. In general, **bold face** are used to denote vectors, *italics* are used to denote scalars or vector magnitude, and though not listed below the use of subscripts x, y, and z denote vector components in those directions.

- A* Cross-sectional area; (6.446 in<sup>2</sup> [41.59 cm<sup>2</sup>] for Major League baseballs).
- C<sub>D</sub>* Coefficient of drag; (non-dimensional).
- C<sub>L</sub>* Coefficient of lift; (non-dimensional).
- C<sub>Y</sub>* Coefficient of cross-force; (non-dimensional).
- D** Drag component of aerodynamic force; (N).
- D* Magnitude of drag component of aerodynamic force; (N).
- d* Diameter of the ball; (2.864 inches [7.26 cm] for Major League baseballs).
- F** Total force vector acting on the ball;  $\mathbf{F} = \mathbf{F}_A + \mathbf{F}_G$ . (N).
- F<sub>A</sub>** Aerodynamic force acting on the ball;  $\mathbf{F}_A = \mathbf{L} + \mathbf{D} + \mathbf{Y}$  (N).
- F<sub>G</sub>** Force due to gravity; (N).
- G* Center of mass of the ball; (m).
- g** Gravitational field strength; (m/s<sup>2</sup>).
- H<sub>G</sub>** Angular momentum with respect to the center of mass of the ball; (kg-m<sup>2</sup>/s).
- I<sub>G</sub>** Inertia with respect to the center of mass of the ball; (kg-m<sup>2</sup>).
- k* Proportionality constant for lift coefficient. (non-dimensional).
- L** Lift component of aerodynamic force; (N).
- L* Magnitude of lift component of aerodynamic force; (N).
- M<sub>G</sub>** Aerodynamic moment with respect to the center of mass of the ball; (N-m).
- n.d.* No date.
- P** Linear momentum; (kg-m/s).
- Re* Reynolds number; (non-dimensional).

## ABBREVIATIONS (Cont.)

- $r$  Radius of the ball; (1.432 inches [3.63 cm] for Major League baseballs).
- $S$  Spin parameter;  $S = U/V$  (non-dimensional).
- $SRD$  Spin Rate Decay parameter; (non-dimensional).
- $t$  Time; (sec).
- $U$  Tangential velocity of the ball ( $r$

## ABSTRACT

In this dissertation the aerodynamic force and initial conditions of pitched baseballs are estimated from high-speed video data. Fifteen parameters are estimated including the lift coefficient, drag coefficient and the angular velocity vector using a parameter estimation technique that minimizes the residual error between measured and estimated trajectories of markers on the ball's surface and the center of mass of pitched baseballs. Studies are carried out using trajectory data acquired from human pitchers and, in a more controlled environment, with a pitching machine. In all 58 pitch trajectories from human pitchers and 20 pitching machine pitches with spin information are analyzed. In the pitching machine trials four markers on the ball are tracked over the first 4 ft (1.22 m) and the center of mass of the ball is tracked over the last 13 ft (3.96 m) of flight.

The estimated lift coefficients are compared to previous measured lift coefficients of Sikorsky (Alaways & Lightfoot, 1998) and Watts & Ferrer (1987) and show that significant differences exist in the lift coefficients of two- and four-seam curve balls at lower values of spin parameter,  $S$ . As  $S$  increased the two- and four-seam lift coefficients merge becoming statistically insignificant. The estimated drag coefficients are compared to drag coefficients of smooth spheres and golf-balls and show that these data sets bound the drag-coefficient of the baseball. Finally, it is shown that asymmetries of the ball associated with the knuckleball can influence the trajectory of the more common curve and fastball.